

ADVANCED UNDERGRADUATE COURSES

Advanced undergraduate courses

- > Calculus and Introduction to Partial Differential Equations (8 ECTS)
- > Analysis and applications (7 ECTS)
- > Introduction to probability theory (4 ECTS)
- > Introduction to optimization (4 ECTS)

Basic graduate courses

- > Operations research (7 ECTS)
- > Stochastic processes and applications (7 ECTS)
- > Numerical statistics & data analysis (4 ECTS)

Advanced undergraduate/basic graduate courses

- > Project (8 ECTS)

ADVANCED UNDERGRADUATE COURSES

CALCULUS AND MODELING OF PARTIAL DIFFERENTIAL EQUATIONS

[Etienne Bernard and Tony Lelievre]

Objectives

This course is constituted of two parts. The first part will consist of the review and consolidation of mathematical techniques for applications in scientific computing: matrix calculus, limited development, ordinary differential equations, and integration.

The second part will be an overview of PDEs as modeling tools in physics and social sciences.

More precisely, the following points will be treated:

- > Ordinary Differential Equations (modeling, existence and uniqueness in the Lipschitz case, Euler schemes).
- > Poisson Equation (modeling, fundamental solutions of the Laplacian, properties of harmonic functions, Green's function for the Dirichlet problem, discretization by finite differences).
- > Heat equation (modeling, Green's function, qualitative properties, discretization, and CFL condition).

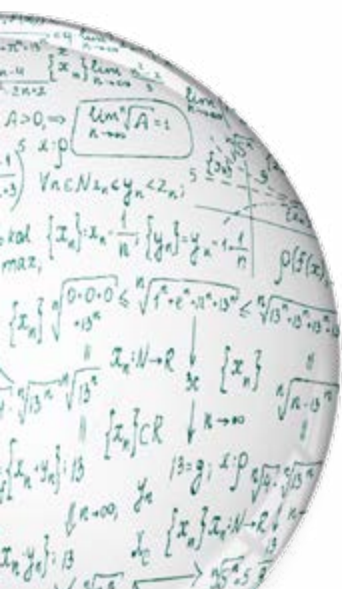
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Modalities

- > For the first part, 6 3-hours sessions of exercices. For the second part, 6 3-hour sessions (1 hour lecture + 1 hour 45 minutes tutorial).
- > complementary study of various topics based on standard textbooks such as Evans, «Partial Differential Equations» (autonomous work supervised by a member of the teaching staff).

Assessment

- > Two written exams of 1h.
- > For the complementary individual project: written report of about 20 pages



ADVANCED UNDERGRADUATE COURSES

ANALYSIS AND APPLICATIONS

[Virginie Ehrbacher]

The aim of this module is to introduce the fundamental mathematical concepts necessary for solving partial differential equations. In this module, we will present the notions of Lebesgue spaces, distribution theory, Fourier series and transforms, and their application to the solution of partial differential equations. More precisely, the following classical topics in analysis will be studied: Banach and Hilbert spaces, measure theory, Lebesgue spaces, Fourier series and transform, distribution theory.

Modalities

- > The majority of the sessions will be lecture/DD sessions. Two independent sessions will be devoted to revision of the solution of ordinary linear differential equations and the calculation of integrals using the change of variables formula. Another TPA session will be devoted to revision.
- > complementary study of various topics based on standard textbooks such as Rudin, «Real and Complex Analysis» (autonomous work supervised by a member of the teaching staff).

Assessment

- > For the lectures: two exams (6 points each), periodic quizzes (4 points) and homeworks (4 points)
- > For the complementary individual project: written report of about 20 pages.

References

A handout in English is available, as well as the slides (in French) corresponding to each class session. Examination papers from previous years and their answers are available on the course's edunet page.

ADVANCED UNDERGRADUATE COURSES

INTRODUCTION TO PROBABILITY THEORY

[Julien Guyon]

Objectives

- > General overview: The aim of this course is to provide engineers with essential knowledge in probability. We will present fundamental concepts (probability space, random variable, distribution, expected value, etc.) as well as common probability distributions. Emphasis will be placed on the tools for characterizing and calculating these distributions. We will introduce the various concepts of convergence to ensure a thorough understanding of the two fundamental theorems: the strong law of large numbers and the central limit theorem. Finally, we will address a more numerical and practical aspect by presenting the main algorithms for simulating random variables and introducing the Monte Carlo method.
- > Academic rationale: Probability theory is the foundation upon which all studies (probabilistic and statistical) involving chance are based. It is used in numerous fields of engineering science (mechanics, physics, chemistry, biology, signal processing, economics, finance, insurance, uncertainty quantification, simulation of rare events, etc.). Probability is also sometimes very useful for solving deterministic problems, as demonstrated by the Monte Carlo method.
- > Professional relevance: Today more than ever, in the era of the artificial intelligence revolution (which is based on statistical machine learning), it is crucial that every engineer be comfortable with the essential concepts (which are probabilistic in nature) of statistical data processing, whatever the data may be.

Modalities

- > Total hours: 18h + exam 3h ; Breakdown CM TD TP: 8h30 - 8h30 - 1h.
- > Complementary study on the use of probabilistic models in engineering or biology based on chapters of the textbook «Modèles aléatoires» in reference below (autonomous work supervised by a member of the teaching staff).

Assessment

- > For the lectures : final exam + bonus points (between 0 and 3) for participation in class and written answers to optional exercises, taken into account only for students who have less than 10 on the examination (with grade capped to 10)
- > For the complementary study, written report of about 15 pages

References

- > Documentary resources: Benjamin Jourdain, Probabilités et statistique, Ellipses, 2009, available at <http://cermics.enpc.fr/~jourdain/probatat/poly.pdf>
- > Module website address: A webpage containing teaching materials and course information will be maintained on the website <http://cermics.enpc.fr/~guyon>.

BASIC GRADUATE COURSES

STOCHASTIC PROCESSES AND APPLICATIONS

[Jean-François Delmas]

Objectives

The course Stochastic Processes and Applications (SPA) presents models in discrete time and discrete state space such as Markov chains, with in particular a focus on their long time behaviors. It gives also an introduction to continuous models such as Brownian motion. An important rôle is played by conditional expectation and martingale theory. Some examples will illustrate how the concepts developed in this course can be used to solve problems from physics, biology, or engineers sciences. The course will be dispensed in French, but the documents will be in English.

All the courses will contain a presentation of the main theory and results with some proofs and some illustrative examples with dedicated exercises.

Modalities

- > 11 sessions of 3h from the end of September to February, including exercises sessions.
- > Complementary study of various topics based on standard textbooks such as «Markov Chains» by Douc, Moulines, Priouret and Soulier or «Markov Chains and Stochastic Stability» by Meyn and Tweedie.

Assessment

- > For the lectures: final exam, periodic quizzes and homeworks
- > For the complementary study, written report of about 20 pages.

References

- > Dedicated lecture notes in English as well as further references are available on the course's webpage.

BASIC GRADUATE COURSES

NUMERICAL STATISTICS & DATA ANALYSIS

[Andrew McRae]

Objectives

General overview:

It is a basic statistics course that provides students with essential statistical tools to make practical inferences from the data they will encounter.

Academic rationale:

This statistics course is intended for all the students of the school. It is a «school mandatory» course. Training all students in statistics is a goal of the school and of the CTI.

Professional relevance:

Statistics have become indispensable in all engineering professions today. The analysis, understanding, and processing of data have become essential in all economic sectors.

Course programme:

The aim of this course is to give students some essential statistical tools to make practical inferences from the data they will encounter. At the end of this module, students should be able to:

- > Design estimators for statistical parameters.
- > Compute and understand confidence intervals.
- > Conduct and interpret statistical tests.

Modalities

- > 5 sessions lecture (1h30) + exercises (1h30), and 1 exam session. No homework.

Assessment

- > Evaluation criteria: Practical work (40%), Exam (60%).
- > For the complementary study, written report of about 20 pages.

BASIC GRADUATE COURSES

OPERATIONS RESEARCH

[Axel Parmentier]

Objectives

Operations research is the discipline of applied mathematics which provides decision support tools. In the industry, decision makers have typically too many possible choices to consider each of them individually. Operations research gives solutions to find a good choice, or even the best choice. Example of applications include route choice, network design or scheduling. Operations research is a must-have in the toolbox of engineers who solve resource allocation problems. This is notably the case of those working in supply-chain, network industries, infrastructure management, finance or information technology architecture. Big data opened the door to a huge number of new applications, and there are many industries where operations research is still underexploited.

At the end of the course, students will master the fundamentals of operational research: the ability to identify a problem that can be addressed by operations research, to model it as a mathematical problem, to propose relevant solution algorithms, and to evaluate the relevance of algorithms and the solutions they return. To that purpose, they will master the main mathematical tools of operations research (Sessions 1 to 6) and their application to an industrial problem (Sessions 7 to 12).

Modalities

- > 12 sessions of 2h45, among which 6 are in small groups, 4 in full group, 2 at home.
- > Complementary study on a theoretical topic or an application of operations research.

Assessment

- > One project (2/5 of the final grade) and two exams (3/5 of the final grade).
- > For the complementary study, written report of about 10 pages (+ code if an application of operations research).

References

Dedicated lecture notes are provided.

ADVANCED UNDERGRADUATE/BASIC GRADUATE COURSES

PROJECT

Objectives

The aim of this semester-long project is to complement the student's training at École des Ponts, both on theoretical aspects and numerical ones. Possible topics include:

- > mathematical and numerical analysis for models in quantum physics.
- > variational methods in physics (Lagrangian and Hamiltonian dynamics, relativity theory, etc).
- > control of dynamical systems.
- > functional inequalities in probability theory.
- > longtime properties of Hamiltonian dynamics and their discretization.
- > nested and Multilevel Monte-Carlo methods for conditional expectations.
- > statistical modeling of random phenomena in physics, biology, finance...
- > Markov Chain Monte-Carlo method and applications in computer science or data analysis.
- > uncertainty propagation and quantification in industrial applications.
- > implement, optimize, test and compare an algorithm published in a scientific journal or in conference proceedings in the fields of image processing or computer vision (ideally, the code should be of sufficient quality to consider a

submission to the Image Processing On Line journal, IPOL, <http://www.ipol.im/>, a journal dedicated to reproducible research in image processing where each article is accompanied by open source, peer-reviewed code and an online demonstration system running directly in the user's browser without any extension).

- > solving an optimization problem motivated by a concrete application from industrial engineering or transportation (modeling, construction of an algorithm, experimentation).
- > Students are welcome to suggest topics to study, or directions they want to emphasize.

Modalities

- > Regular meetings with a senior researcher and weekly meetings with a junior researcher.

Assessment

- > Written report and script of the code used for numerical simulations (if relevant).