École des Ponts
ParisTech

**Syllabus 2022**
« PAM'S » PARIS APPLIED MATHS SEMESTER
@ Ecole des Ponts ParisTech

# ADVANCED UNDERGRADUATE COURSES

## ANALYSIS AND APPLICATIONS

[Antoine Levitt and Gabriel Stoltz]

This lecture provides an introduction to various domains of 20th century analysis, with some emphasis on analytical or numerical techniques allowing to better understand and solve the partial differential equations which are encountered in various scientific domains (transport equation, heat equation, Schrodinger equation, wave equation, etc).

On the theoretical side, the lectures first introduce basic elements of functional and applied analysis, including operators on Banach spaces; and then Lebesgue spaces, which build upon the theory of Lebesgue integrals and measure theory (whose aim is to generalize the notion of Riemann integrals).

The theory of distributions, which allows to give a meaning to very singular solutions to partial differential equations, is next presented.

Useful tools such as the Fourier transform are also discussed. Finally, numerical methods to actually solve partial differential equations are constructed, either through spectral approaches based on Fourier series, or by finite difference methods. In both cases, hands-on sessions allow the students to witness the methods at play.

### Modalities

> 3 sessions of 3h at the beginning of September, then 12 sessions of 2.5h, taught using «flipped classrooms» (the content of the course has to be learned at home beforehand, while sessions with the teachers are devoted to discussions on subtle points of the theory, and exercices in groups of 3-4 students).

> complementary study of various topics based on standard textbooks such as Rudin, «Real and Complex Analysis» or Evans, «Partial Differential Equations» (autonomous work supervised by a member of the teaching staff).

### Assessment

> For the lectures: two exams (6 points each), periodic quizzes (4 points) and homeworks (4 points).

> For the complementary individual project: written report of about 20 pages.

### References

Dedicated lecture notes in English are provided and further references are given on the course's webpage.

École des Ponts ParisTech

# ADVANCED UNDERGRADUATE COURSES

## INTRODUCTION TO PROBABILITY THEORY

[Aurélien Alfonsi]

### Objectives

This course provides the essential knowledge in probability for undergraduate students. The fundamental notions (probability space, random variable, law of a random variable, expectation,...) as well as the usual probability laws with real and integer values are presented. Emphasis is put on providing the tools to characterize and calculate probability laws. The various notions of convergence are discussed, which allows to fully understand the statements of two fundamental convergence theorems: the strong law of large numbers and the central limit theorem. Finally, classical numerical methods to simulate random variables are described, and an introduction to Monte-Carlo methods is given.

Detailed content of the lectures:

> Space of probability, random variables, law, expectation. Link to the preparatory class program when the probability space is countable.

> Real random variables, usual probability laws with densities, probability laws neither discrete nor with density, calculation of the probability law by using test functions, characteristic function (link with Laplace and generating function).

> Random variables with vectorial values, dependency, Gaussian vectors, calculation of the law by the test functions (Jacobian), characteristic function.

> Presentation of the different convergence modes (in probability, almost sure, in law, L^p). Examples and counter-examples.

> Strong Law of Large Numbers and Central Limit Theorem. Construction of confidence intervals.

> Simulation of random variables. Inverse transform sampling. Box-Muller. Simulation of a Gaussian vector.

> Monte Carlo method. Interest of the method, curse of the dimension. Some examples, and variance reduction.

### Modalities

> 7 sessions of 3h; lecture and exercice sessions are all done in classroom format with no more than 30 students. Each teacher will suggest to the students (at least twice) to solve optional exercises for the next lecture. One or two self-study sessions (kind of office hours) are planned. They will make it possible to give priority to student support (questions/ answers about the course and the exercises done in class) and possibly complementary exercises.

> complementary study on the use of probabilistic models in engineering or biology based on chapters of the textbook «Modèles aléatoires» in reference below (autonomous work supervised by a member of the teaching staff).

### Assessment

> For the lectures : final exam + bonus points (between 0 and 3) for participation in class and written answers to optional exercises, taken into account only for students who have less than 10 on the examination (with grade capped to 10).

> For the complementary study, written report of about 15 pages.

### References

Benjamin Jourdain, Probability and Statistics, Ellipses, 2009.

Benjamin Jourdain and Jean-François Delmas, Modèles aléatoires : applications aux sciences de l'ingénieur et du vivant, Springer, 2006.

A web page containing teaching materials and course information is maintained http://cermics.enpc.fr/~alfonsi/Proba1A.html

ADVANCED UNDERGRADUATE COURSES

# INTRODUCTION TO OPTIMIZATION

[Frédéric Meunier]

## Objectives

The main objective of this course is to introduce the students to the classical methods for solving elementary problems of continuous optimization in finite dimension, in particular the KKT conditions and the simplex algorithm. The secondary objectives are to show the variety of application areas (other areas of mathematics, industrial engineering, economics, ...) and to give a first experience of modeling for decision problems.

## Modalities

> Each session starts by a 50-minute lecture in the lecture hall and then continues with a tutorial of 2h15 in small groups. There are two extra sessions: one to introduce a short project to be done during the course; one to prepare to the final exam.

> Complementary study of various topics based on standard textbooks, for instance on calculus of variations or algorithms such as the Frank-Wolfe method.

## Assessment

> For the lectures: An exam (2/3 of the final mark) and a short project (1/3 of the final mark).

> For the complementary individual project: written report of about 20 pages.

## References

Lecture notes are provided.

École des Ponts
ParisTech

ADVANCED UNDERGRADUATE COURSES

# INTRODUCTION TO PROGRAMMING

[Pascal Monasse]

## Objectives

Students are supposed to have followed an introductory course on programming in one of the following languages: C++ (ideally), C or Java. This course will provide some basic notions of algorithmics, that will be applied to the development of a project programmed in C++.

The first part of the lectures introduces standard data structures, such as vector, list, stack, queue, priority queue, and discusses the complexity of their operations and the possible implementations. Some important principles of efficient algorithms will be exposed, such as divide and conquer and dynamic programming. They are applied in practical sessions including classical sorting algorithms, Fast Fourier Transform and fast marching for solving a class of partial differential equations. Applications include image processing (copy/paste between images, contrast enhancement) and computation of geodesics.

The second part of the lectures is an assisted project developed by groups of 2 or 3 students. Many subjects are possible, for example strategic games, video games, computer graphics, image processing. Code versioning and sharing with git will be introduced and more generally usage of programming tools will be presented. The end result should be a quality C++ code implementing a working program, which will be presented in front of the whole class in a short oral and visual demonstration.

## Modalities

> 6 sessions of 2.5h, including 4h of formal lectures the remaining time consists in practical sessions where C++ code must be completed and finished as homework; 6 sessions of assisted project development; 2 optional sessions of 2.5h on remainder of basics of C++ programming for students who need it.

> topics for complementary work: study of an algorithm (correctness, theoretical complexity, practical performance), examination of significant recent additions to the C++ language (what needs they address), or topics in high performance computing (such as memory cache efficiency, vectorized instruction sets, parallelism).

## Assessment

> For the lectures: practical session C++ codes (4), written exam (1), project code and oral defense (1).

> For the complementary individual project: either a written report of about 20 pages, or a shorter report of 5 pages complemented by some source code implementing the studied subject.

## References

Lecture notes are provided.

École des Ponts ParisTech

BASIC GRADUATE COURSES

# STOCHASTIC PROCESSES AND APPLICATIONS

[Jean-François Delmas]

## Objectives

Markov processes are widely used to model stochastic evolution of dynamical systems. In this lecture we consider the simplest case of such processes: the so-called discrete time Markov chains (taking values in a finite or countable state space). We study the corresponding ergodic theorems, that is, the asymptotic convergence in large time of the Markov chains towards a stationary distribution. We also introduce martingales (and stopping times and the optional stopping theorem), as well as related convergence theorems, to further characterize the properties of Markov chains. One session is devoted to a detailed approach of the optimal stopping theory (which is the existence and characterization of the best time if any to stop a game in order to maximize a reward). We eventually give in the last sessions a brief introduction to the Brownian motion and stochastic Wiener integrals.

The exercises sessions as well as the exams with correction (available on the web page https://cermics.enpc.fr/~delmas/Enseig/proba2.html) provide other various examples of applications of the Markov chains and martingales theories (in biology, physics, mathematics, ...).

## Modalities

- For the lectures: final exam, periodic quizzes and homeworks.
- For the complementary study, written report of about 20 pages.

## References

Dedicated lecture notes in English as well as further references are available on the course's webpage.

# BASIC GRADUATE COURSES

## APPLIED STATISTICS / MATHEMATICAL STATISTICS

[Vincent Lefieux and Julien Reygner]

### Objectives

These lectures provide an introduction to the practical methods of statistics and data analysis and their mathematical foundations. In statistics, the topics covered are parametric estimation, confidence intervals and hypothesis testing. In data analysis, principal component analysis, clustering, linear and logistic regression are addressed.

Depending on students' background and interest, two courses are available: an «applied statistics» course oriented toward methods and applications, and a «mathematical statistics» course with a deeper focus on theoretical aspects.

### Modalities

> 12 sessions of 2h including lectures, exercises, hands-on tutorials and short group projects.

> complementary individual project based on applied or theoretical textbook.

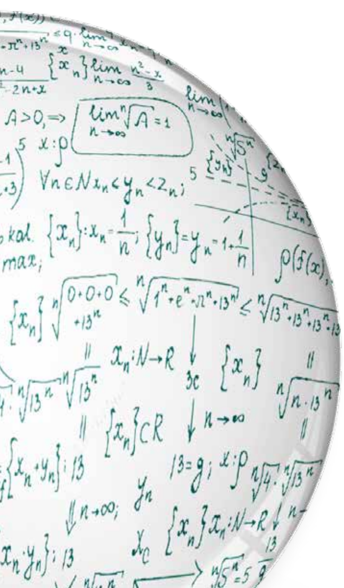
### Assessment

> for the course: written exam, homework exercises and/or group project ;

> for the complementary individual project: written report.

### References

Dedicated lecture notes in English are provided and further references are given on the course's webpage.



École des Ponts ParisTech

BASIC GRADUATE COURSES

# OPERATIONS RESEARCH

[Axel Parmentier]

## Objectives

Operations research is the discipline of applied mathematics which provides decision support tools. In the industry, decision makers have typically too many possible choices to consider each of them individually. Operations research gives solutions to find a good choice, or even the best choice. Example of applications include route choice, network design or scheduling. Operations research is a must-have in the toolbox of engineers who solve resource allocation problems. This is notably the case of those working in supply-chain, network industries, infrastructure management, finance or information technology architecture. Big data opened the door to a huge number of new applications, and there are many industries where operations research is still underexploited.

At the end of the course, students will master the fundamentals of operational research: the ability to identify a problem that can be addressed by operations research, to model it as a mathematical problem, to propose relevant solution algorithms, and to evaluate the relevance of algorithms and the solutions they return. To that purpose, they will master the main mathematical tools of operations research (Sessions 1 to 6) and their application to an industrial problem (Sessions 7 to 12).

## Modalities

> 12 sessions of 2h45, among which 6 are in small groups, 4 in full group, 2 at home.

> Complementary study on a theoretical topic or an application of operations research.

## Assessment

> One project (2/5 of the final grade) and two exams (3/5 of the final grade).

> For the complementary study, written report of about 10 pages (+ code if an application of operations research).

## References

Dedicated lecture notes are provided.

École des Ponts ParisTech



# ADVANCED UNDERGRADUATE/BASIC GRADUATE COURSES

## PROJECT

### Objectives

The aim of this semester-long project is to complement the student's training at Ecole des Ponts, both on theoretical aspects and numerical ones. Possible topics include:

> mathematical and numerical analysis for models in quantum physics

> variational methods in physics (Lagrangian and Hamiltonian dynamics, relativity theory, etc)

> control of dynamical systems

> functional inequalities in probability theory ;

> longtime properties of Hamiltonian dynamics and their discretization

> nested and Multilevel Monte-Carlo methods for conditional expectations

> statistical modeling of random phenomena in physics, biology, finance...

> Markov Chain Monte-Carlo method and applications in computer science or data analysis ;

> uncertainty propagation and quantification in industrial applications

> implement, optimize, test and compare an algorithm published in a scientific journal or in conference proceedings in the fields of image processing or computer vision (ideally, the code should be of sufficient quality to consider a submission to the Image Processing On Line journal, IPOL, http://www.ipol.im/, a journal dedicated to reproducible research in image processing where each article is accompanied by open source, peer-reviewed code and an online demonstration system running directly in the user's browser without any extension)

> solving an optimization problem motivated by a concrete application from industrial engineering or transportation (modeling, construction of an algorithm, experimentation)

> Students are welcome to suggest topics to study, or directions they want to emphasize

### Modalities

> Regular meetings with a senior researcher and weekly meetings with a junior researcher.

### Assessment

> Written report and script of the code used for numerical simulations (if relevant).